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BOND VALUES

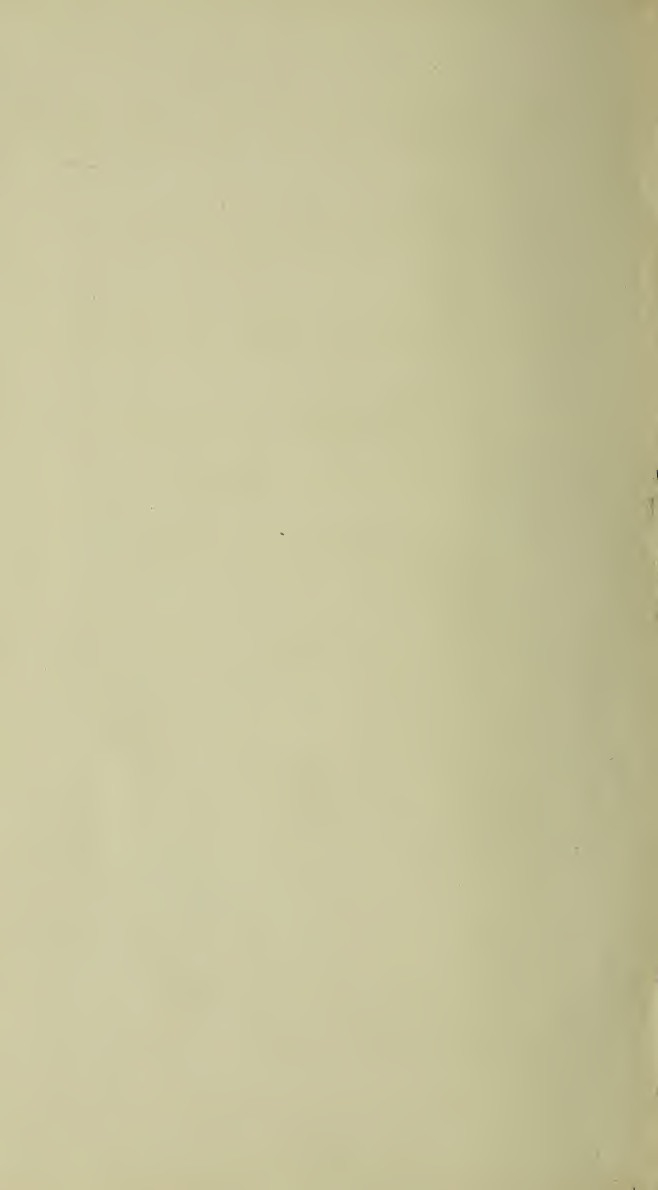
UNUSUAL COUPON RATES

ARTHUR S. LITTLE

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~~Economics~~



FORMULAE

For Obtaining from Ordinary
Bond Tables

VALUES FOR BONDS

At Various Unusual
Coupon Rates

From 3.01% to 6.50%

2068

Devised and Publisht by
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303 N. Fourth Street, St. Louis

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Preface.

Up to 1902 the bond tables then in use seem to have met all requirements, but during that year the Odd Rate problem arose, * and has been a live question ever since. Publishers have been kept busy issuing enlarged or special editions providing for bonds at $4\frac{1}{4}\%$, $5\frac{1}{2}\%$, etc., but it is hopelessly out of the question to furnish comprehensive tables for all the odd rate bonds that exist today;—much less the others that are apt to come. For there is said to be a growing tendency on the part of cities, school districts, etc., to sell new issues at par net and have the competition between buyers take place in the coupon rate instead of the price. This plan, in spite of whatever inconvenience it may occasion bond dealers, investors and accountants, is, in my opinion, unquestionably the wisest and best from the standpoint of political economy and municipal accounting, for a municipal corporation is not supposed to have a Capital or Profit & Loss account, † and if the coupons really represent, or rather, are equivalent to, the exact interest on the outstanding bonds, it is quite possible, in conjunction with the Serial Bond feature, skillfully applied, to provide for an equitable system of taxation and budget-making that is almost ideal. Believing, therefore, that at no distant date there will be frequent issues of municipal bonds at such rates as 3.95% , 6.35% , etc., or even 3.94% or 6.37% , I have devised the accompanying table of formulae by which values for any odd rate may be readily obtained by adding together a certain number of times a certain number of values found in the ordinary tables in use.

These formulae may be applied to any table, regardless of how frequently the coupons mature, or to what time unit standard the rate of income is referred.

As work of this nature will almost invariably be performed in an office, the use of an adding machine is of course contemplated, in which case it is merely a matter of adding, and multiplying does not enter into the operation (except effectively).

*A large issue of St. Louis World's Fair bonds at $3\frac{1}{4}\%$, which caused quite a rumpus in investment circles and occasioned the publication of a special table to cover the case.

† "Its principal asset is its power of confiscating the property of its members and others within its limits, thru taxation, to an extent which cannot be valued, but which is measured by the needs, as legally ascertained, of its members.....hence its balance sheet is non-existent. The highest function of municipal bookkeeping is the coordination of revenue and expenditure, of sacrifice and service."

THE PHILOSOPHY OF ACCOUNTS. Sprague.

359494

It is also best, when practicable, to work with an assistant; one party operating the machine while the other dictates the steps to be taken, and it will no doubt be found that a fairly comprehensive set of manuscript tables can be turned out in this manner in a surprisingly short time. However, even when an adding machine is not available, this method is always feasible with pencil & paper, and in many cases at least will prove easier, quicker and safer than any other.

The mode of procedure is simplicity itself, and the items in the schedule simply represent the number of times the value for the bond indicated at the head of the column is to be written, or rather taken.

The total is always the value desired.

The results are always increased tenfold or a hundredfold, and therefore one or two final figures are worthless, and must be rejected. In other words, the interpolated value obtained is never (except by accident) accurate to a greater number of places than the tables that were employed. The rejected figures may, however, and in fact should be utilized to correct to the nearest unit the final figure that is retained.

The task of concocting these formulae did not seem to admit of any well defined system and was more in the nature of a gigantic jigsaw puzzle, and of course proved very laborious and trying. Furthermore, it was necessary to produce results in a form as equally well adapted as possible for every type of adding machine in use. Had there been no necessity for consideration of any other machine than the ten-key type with visible printing, many of the formulae could have been given in a much simpler form.

Nevertheless, the entire table has been carefully verified twice by independent and totally different methods. No less conscientious care will be exercised in examining the printer's proofs; therefore the table is believed to be entirely correct throughout.

ARTHUR S. LITTLE

St. Louis, August 10, 1915.

EXAMPLES.

Find a 3.65% basis for a 3.65% bond, 19 years to run.

	9795 74
	9795 74
	9795 74
	9795 74
3½% Bond taken 9 times	9795 74
	9795 74
	9795 74
	9795 74
	9795 74

5% Bond taken 1 time 11838 37

100000 03

This problem is of course an idle one, and was purposely selected in order to obtain a glaringly correct result. The above is representativ of the uniform accuracy that is always obtaind.

Find an 0% basis for a 4.77% annual bond, 1 year to run.

3½% bond 2 times.....{1035 00
1035 00

4½% bond 80 times.....{10450 00
10450 00
10450 00
10450 00
10450 00
10450 00
10450 00

5% bond 8 times.....{1050 00
1050 00
1050 00
1050 00
1050 00
1050 00
1050 00

7% bond 10 times.....10700 00

104770 00

Like the foregoing, this is an impractical problem. It is interesting, however, and also suggests an excellent and simple test of the accuracy of any formula.

Find a 4% basis for a 4.37% bond, 6 months to run.

	{	9950 98
		9950 98
		9950 98
		9950 98
3% bond 9 times.....	{	9950 98
		9950 98
		9950 98
		9950 98
		9950 98

	{	100245 10
		100245 10
		100245 10
		100245 10
4½% bond, 90 times.....	{	100245 10
		100245 10
		100245 10
		100245 10
		100245 10

5% bond, 1 time.....10049 02

1001813 74

Result, cut down100.1814

PROOF.

Add 6 months' interest at 4%..... 2.0036

Value of matured bond & coupon.....102.1850

Find a 6% basis for a $4\frac{5}{8}\%$ bond, 10 years to run.

$3\frac{1}{2}\%$ bond, 1 time.....8140 32

	88841 90
	88841 90
	88841 90
	88841 90
$4\frac{1}{2}\%$ bond, 90 times.....	88841 90
	88841 90
	88841 90
	88841 90
	88841 90

6% bond, 9 times.....90000 00

897717 42

Which cuts down to.....89.7717

PROOF

Add 6 months' interest at 6%.....	2.6932
	<hr/> 92.4649

Subtract coupon.....	2.3125
	<hr/>

Derived value for $9\frac{1}{2}$ years.....	90.1524
---	---------

agreeing with result obtained directly, as shown below.

8209 52

89257 10
89257 10
89257 10
89257 10
89257 10
89257 10
89257 10
89257 10
89257 10
89257 10

90000 00

901523 42

SUPPLEMENTARY REMARKS

Altho the method of interpolation provided for by the accompanying formulae is believed to be thoroly original and superior to any other yet devised for the purpose, at the same time all the elements of novelty are strictly mechanical in their nature, and the groundwork of the system is the abstract general law about to be noted. A mere knowledge of the existence of this relationship that bond values at different coupon rates bear to each other is unfortunately much less widespred among bond dealers and investors than it should be, and still less prevalent is an intelligent comprehension of the significance of this law, and the various ways in which it may be invoked in the solution of sundry practical problems; hence it is deemed worth while to explain briefly why all these different bond values are governd by this law.

If we take, say, the 3% income line on the 10 year page of Rollins' or Deguhee's 4-place bond tables and compare the values given we will find that they differ from each other as follows:

7% Bond	134.34	8.59
6% Bond	125.75	8.58
5% Bond	117.17	8.59
4% Bond	108.58	8.58
3% Bond	100.00	

Further similar experiments continued indefinitely for any rate of income or any time to run always produce results of the same character. This is highly suggestiv, but is not proof. Better still, it may be an approximation that holds good for short values only. We therefore try Deguhee's 6-place table, obtaining:

134.3373	8.5843
125.7530	8.5844
117.1686	8.5843
108.5843	8.5843
100.0000	

and finally Sprague's 8-place table:

134.337278	8.584320
125.752958	8.584319
117.168639	8.584320
108.584319	8.584319
100.000000	

Results of a similar nature are yielded by:

The tables of the writer for annual bonds on a semi-annual basis;

The tables of Rollins for annual bonds on an annual basis;

The low rate tables of Sprague for quarterly bonds on a semi-annual basis;

The tables of various authors for quarterly bonds on a quarterly basis.

In view of all of this it is impossible to do else than conclude that the bond tables in use are governd by a very simple and inflexible law.

Inasmuch as we have just observd that by starting with the value for a 3% bond and successivly adding 8.584320 we obtaind values for bonds at 4%, 5%, 6% & 7%, we may safely conclude that this process may be continued indefinitely, thus suggesting at once that any bond table contains (potentially) values for bonds at such coupon rates as 8%, 9%, 10%, etc.

It is also evident that insted of starting with the 3% bond and building up by addition we may start with the 7% bond and tear down by subtraction. This process, after passing the scope of the tables produces:

2% Bond 9 1 4 1 5 6 8 0

1% Bond 8 2 8 3 1 3 6 0

and finally

0% Bond 7 4 2 4 7 0 4 0

A 10 year s. a. bond carrying coupons for 0% is manifestly nothing more than a promis to pay a single sum 10 years hence, and the present worth of Unity, according to the tables of Reussner, is 7 4 2 4 7 0 4 1 8, tallying exactly with the value for an 0% bond just obtaind.

Reussner's tables also contain a list of values known as the Present Worth of an Annuity of Unity per half-year. For instance, at the 3% rate, 10 years, the value given is 1 7 . 1 6 8 6 3 8 7 8 5 . This means that; placing upon money a value of 3%,

compounded semi-annually, then a salary, rental, etc., of Unity per 6 months for 10 years (total face value 20) is worth today 17 . 1686 etc.

The present worth of $\frac{1}{2}$ of Unity is of course $\frac{1}{2}$ of the above, or 8.584319392, which corresponds exactly with the constant difference that we found to exist between the 3%, 4%, 5%, etc., bonds.

The facts in the case, therefore, are as follows, despite what popular misapprehensions or perverted conceptions may exist:

A bond is a promise to pay a single large sum at a fixed time, accompanied by a chain of promises to pay smaller sums * for uniform amounts at fixed times.

The par value of the bond is the aggregate face value of these numerous promises to pay fixed sums at specified times. For example, the par value of a 20 year 5% bond is 200.

Investment in a bond for gain consists in discounting, at some rate of interest, compounded with some standard of frequency, these various promises to pay fixed sums at specified times.

Or, putting it another way, an investment for gain is merely the old story of buying cheap and selling dear; the purchase of a 20 year 5% bond at $105\frac{1}{8}$ being simply a case of a man paying \$1051.25 for a stock of goods that he knows with absolute certainty he will retail, during the next 20 years, for exactly \$2,000.

This unorthodox but correct PAR just defined also constitutes the critical or absolute value of any bond, viz, an 0% basis; being the extreme price that an investor may pay without incurring actual loss.† Or, it may be said to be the bond value representing an investment not for gain, but for investment's

*Usually, but not necessarily so. A bond having a coupon rate of 300% could, under certain conditions, be a particularly desirable and expedient form of a loan, and also constitute, at the proper price, an exceptionally suitable investment for certain classes of investors. It would, however, be necessary;

(a) For the investor to regard the price paid as a basic investment value and not a terrifying, heart-rending, soul-racking "premium" that will be "lost."

(b) That both the issuing corporation and the investor keep their books along different lines from the kindergarten methods almost universally practised at present.

† What is commonly known as the "par" of a bond is a PUNCTUAL INTEREST BASIS; a particular and unique form of bond value, whose relations to the countless other basic values that exist bears a striking analogy to the relations of a circle to ellipses in general. This Punctual Interest Basis is far from being merely a normal, natural, logical value for a bond.....one that every one knows.....one that requires no skill to be ascertained, etc., but on the contrary possesses features of considerable importance in philosophical research, political economy, the new school of investment accounting, actuarial computations, etc., which features, however, cannot be discussed in detail here.

sake alone. This of course rarely occurs in bonds, except negatively, when the owners wilfully abstain from collecting them when they become due. An enormous amount of money is locked up in matured United States bonds & coupons and pension checks in this manner. But investment for investment's own sake is practised in numerous ways by people in all walks of life; favorite vehicles being Post Office or Express money orders, bank drafts, car tickets, postage stamps, etc.

The bond values in ordinary use are merely the present worths of:

- { The future payment of the single sum represented by the face of the bond,
- { The symmetrical series of payments represented by the coupons.

Hence any bond table, instead of being a scale of "prices" is merely a complex set of present worth tables, especially adapted for obtaining conveniently and expeditiously the aggregate present worth of the various promises to pay that, taken together, constitute bonds as ordinarily constructed.

It will be seen, therefore, that while the utility of a bond table is greatly enhanced when values for various coupon rates are given, at the same time this feature is a luxury rather than an essential, and all that the unskilled layman absolutely needs, in order to get anything he wants, are:

Tables of the Present Worth of Unity at various times & rates;

Tables of the Present Worth of an Annuity of Unity for various times & rates.

	3%	3½%	4%	4½%	5%	6%	7%
3.01	99	...	1
3.02	99	1
3.03	99	1	...
3.04	99	1
3.05	9	1
3.06	90	9	...	1
3.07	90	8	...	2
3.08	90	8	2
3.09	90	2	8
3.10	9	...	1
3.11	90	...	9	...	1
3.12	90	...	9	1	...
3⅛	90	...	8	1	...	1	...
3.13	90	...	9	1
3.14	90	...	8	2	...
3.15	9	1
3.16	90	8	2
3.17	90	...	3	...	7
3.18	90	...	2	...	8
3.19	90	2	8
3.20	9	1
3.21	90	9	1	...
3.22	90	9	...	1
3.23	90	7	3	...
3.24	90	6	4	...
3.25	90	2	8	...

	3%	3½%	4%	4½%	5%	6%	7%
3.26	90	...	2	8	...
3.27	91	9	...
3.28	90	...	1	9	...
3.29	90	1	9	...
3.30	9	1	...
3.31	90	9	1
3.32	90	8	2
3.33	90	2	8
3.34	90	...	2	8
3.35	3	7
3.36	91	9
3.37	90	...	1	9
3¾	90	1	9
3.38	90	1	...	9
3.39	90	1	9
3.40	2	8
3.41	80	...	9	...	1	10	...
3.42	80	...	10	8	2
3.43	80	...	9	10	1
3.44	80	1	...	9	...	10	...
3.45	1	9
3.46	9	90	1
3.47	9	90	1
3.48	9	90	1	...
3.49	9	90	1
3.51	...	99	...	1

	3%	3½%	4%	4½%	5%	6%	7%
3.52	3	90	7
3.53	2	90	8
3.54	1	90	9
3.55	...	9	1
3.56	...	90	9	...	1
3.57	...	90	9	1	...
3.58	...	90	9	1
3.59	...	91	...	9
3.60	...	9	...	1
3.61	...	90	7	3	...
3.62	...	90	3	...	7
3⅝	...	90	2	1	7
3.63	1	90	9
3.64	...	90	1	...	9
3.65	...	9	1
3.66	...	90	9	1	...
3.67	...	90	9	...	1
3.68	...	90	7	3	...
3.69	2	90	8	...
3.70	...	8	...	2
3.71	...	90	7	...	3
3.72	1	90	9	...
3.73	...	90	1	9	...
3.74	...	90	1	9	...
3.75	...	9	1	...
3.76	...	90	9	1

	3%	3½%	4%	4½%	5%	6%	7%
3.77	...	90	8	2
3.78	...	90	7	3
3.79	...	90	6	4
3.80	2	...	8
3.81	1	90	9
3.82	...	90	1	9
3.83	...	90	1	...	9
3.84	...	90	1	9
3.85	...	3	7
3.86	10	9	80	1
3.87	9	10	80	...	1
3⅞	10	9	80	1	...
3.88	9	10	80	1	...
3.89	9	10	80	1
3.90	1	...	9
3.91	9	...	91
3.92	9	...	90	...	1
3.93	9	...	90	1	...
3.94	9	...	90	1
3.95	...	1	9
3.96	...	9	90	1
3.97	...	8	90	2
3.98	...	7	90	3
3.99	1	...	99
4.01	99	...	1
4.02	99	1	...

	3%	3½%	4%	4½%	5%	6%	7%
4.03	99	1
4.04	...	1	90	9
4.05	9	1
4.06	2	...	90	...	8
4.07	90	6	4
4.08	1	...	90	...	9
4.09	91	...	9
4.10	9	...	1
4.11	90	...	9	1	...
4.12	90	...	9	...	1
4½/8	2	...	90	1	...	7	...
4.13	2	...	90	...	1	7	...
4.14	2	...	90	8	...
4.15	7	3
4.16	90	2	1	7	...
4.17	1	...	90	9	...
4.18	91	9	...
4.19	90	...	1	9	...
4.20	9	1	...
4.21	90	9	1
4.22	90	8	2
4.23	90	7	3
4.24	90	6	4
4.25	...	7	3	...
4.26	1	...	90	9
4.27	91	9

	3%	3½%	4%	4½%	5%	6%	7%
4.28	90	...	1	...	9
4.29	90	1	9
4.30	...	2	...	8
4.31	9	...	11	80
4.32	9	...	10	80	1
4.33	9	...	10	80	...	1	...
4.34	9	...	10	80	1
4.35	1	9
4.36	9	...	1	90
4.37	9	90	1
4¾	5	5	...	90
4.38	9	90	...	1	...
4.39	9	90	1
4.40	...	1	...	9
4.41	...	9	...	91
4.42	3	...	7	90
4.43	...	8	...	90	2
4.44	1	...	9	90
4.45	1	9
4.46	9	90	1
4.47	9	90	...	1	...
4.48	9	90	1
4.49	...	1	...	90
4.51	8	90	2
4.52	3	90	7
4.53	2	90	8

	3%	3½%	4%	4½%	5%	6%	7%
4.54	1	90	9
4.55	9	1
4.56	90	9	1	...
4.57	90	8	2	...
4.58	90	7	3	...
4.59	90	6	4	...
4.60	2	8
4.61	90	4	6	...
4.62	1	90	...	9	...
4.5/8	...	1	...	99	...	9	...
4.63	1	90	...	9	...
4.64	90	1	9	...
4.65	9	...	1	...
4.66	90	...	9	1
4.67	90	...	8	2
4.68	90	...	7	3
4.69	90	...	6	4
4.70	...	2	8
4.71	90	...	4	6
4.72	90	...	3	7
4.73	90	...	2	8
4.74	90	...	1	9
4.75	9	1
4.76	2	80	8	...	10
4.77	...	2	...	80	8	...	10
4.78	2	80	8	...	10

	3%	3½%	4%	4½%	5%	6%	7%
4.79	1	80	9	...	10
4.80	1	9
4.81	9	...	1	...	90
4.82	9	91
4.83	9	90	1	...
4.84	9	90	...	1
4.85	...	1	9
4.86	...	9	...	1	90
4.87	...	8	...	2	90
4.7/8	...	5	5	...	90
4.88	2	...	8	...	90
4.89	1	...	9	...	90
4.90	1	...	9
4.91	9	...	91
4.92	9	...	90	1	...
4.93	9	...	90	...	1
4.94	...	1	...	9	90
4.95	1	9
4.96	8	92
4.97	...	2	98
4.98	1	99
4.99	1	...	99
5.01	99	1	...
5.02	99	...	1
5.03	...	2	1	...	90	7	...
5.04	3	...	90	7	...

	3%	3½%	4%	4½%	5%	6%	7%
5.05	1	8	1	...
5.06	2	...	90	8	...
5.07	1	90	9	...
5.08	1	...	90	9	...
5.09	91	9	...
5.10	9	1	...
5.11	90	9	1
5.12	2	90	...	8
5½/8	1	1	90	...	8
5.13	...	2	90	...	8
5.14	2	...	90	...	8
5.15	2	1	7	...
5.16	1	90	...	9
5.17	1	...	90	...	9
5.18	91	...	9
5.19	90	1	9
5.20	8	2	...
5.21	3	80	7	10
5.22	4	...	80	6	10
5.23	2	80	9	9
5.24	3	...	80	7	10
5.25	...	3	7	...
5.26	2	...	80	8	10
5.27	1	80	9	10
5.28	1	...	80	9	10
5.29	1	20	9	70	...

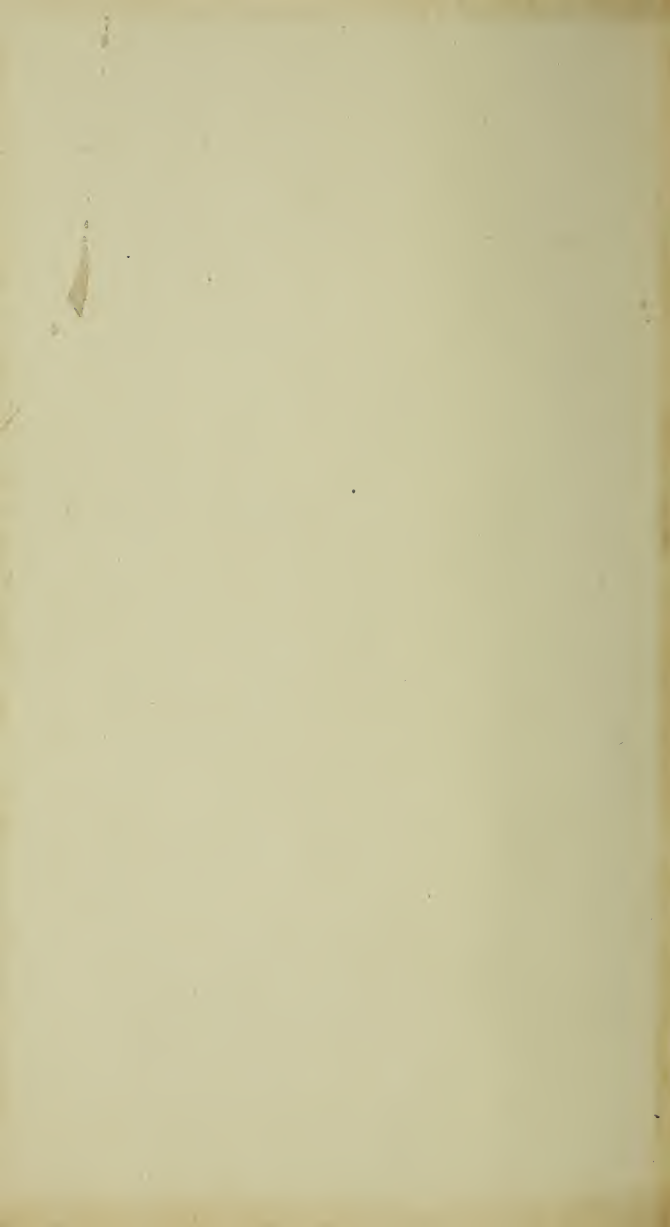
	3%	3½%	4%	4½%	5%	6%	7%
5.30	7	3	...
5.31	...	20	9	...	1	70	...
5.32	2	20	...	8	...	70	...
5.33	...	20	9	70	1
5.34	...	20	2	8	...	70	...
5.35	...	2	...	1	...	7	...
5.36	...	20	...	8	2	70	...
5.37	...	20	3	...	7	70	...
5⅜	...	20	...	5	5	70	...
5.38	1	20	9	70	...
5.39	...	20	1	...	9	70	...
5.40	2	8	...
5.41	9	...	1	20	...	70	...
5.42	2	...	20	8	...	70	...
5.43	...	2	20	8	...	70	...
5.44	...	1	20	9	...	70	...
5.45	1	1	8	...
5.46	9	10	1	80	...
5.47	9	10	1	80	...
5.48	1	...	20	...	9	70	...
5.49	9	10	80	1
5.50	...	2	8	...
5.51	9	...	1	...	20	70	...
5.52	10	...	8	...	2	80	...
5.53	10	...	9	80	1
5.54	1	10	9	80	...

	3%	3½%	4%	4½%	5%	6%	7%
5.55	3	...	7	...
5.56	...	10	9	...	1	80	...
5.57	...	10	8	...	2	80	...
5.58	...	10	7	...	3	80	...
5.59	1	...	9	...	20	70	...
5.60	2	8	...
5.61	...	10	8	80	2
5.62	2	...	10	8	...	80	...
5.5⅘	...	5	10	...	5	80	...
5.63	...	2	10	8	...	80	...
5.64	...	1	10	9	...	80	...
5.65	1	1	...	8	...
5.66	2	...	10	...	8	80	...
5.67	...	2	10	...	8	80	...
5.68	1	...	10	...	9	80	...
5.69	...	4	...	10	6	80	...
5.70	1	9	...
5.71	9	...	1	90	...
5.72	9	1	90	...
5.73	9	91	...
5.74	8	2	90	...
5.75	...	1	9	...
5.76	...	8	2	90	...
5.77	3	...	7	90	...
5.78	8	90	2
5.79	1	...	9	90	...

	3%	3½%	4%	4½%	5%	6%	7%
5.80	1	9	...
5.81	9	...	1	90	...
5.82	9	91	...
5.83	9	90	1
5.84	3	7	90	...
5.85	1	...	9	...
5.86	8	2	90	...
5.87	...	2	8	90	...
5⅞	5	5	90	...
5.88	1	9	90	...
5.89	1	...	9	90	...
5.90	1	9	...
5.91	9	91	...
5.92	9	90	1
5.93	1	...	7	90	2
5.94	8	90	2
5.95	...	3	7
5.96	...	4	90	6
5.97	1	99	...
5.98	1	99	...
5.99	1	99	...
6.01	99	1
6.02	2	90	8
6.03	...	2	90	8
6.04	3	90	7
6.05	...	2	...	1	7

	3%	3½%	4%	4½%	5%	6%	7%
6.06	2	90	8
6.07	1	90	9
6.08	1	90	9
6.09	91	9
6.10	3	7
6.11	9	...	1	20	70
6.12	9	20	1	...	70
6.1⅛	5	5	...	20	70
6.13	9	20	...	1	70
6.14	2	8	...	20	70
6.15	2	1	7
6.16	...	20	2	8	70
6.17	1	20	9	70
6.18	...	20	1	9	70
6.19	...	20	1	9	70
6.20	2	8
6.21	20	...	9	1	70
6.22	20	...	8	2	70
6.23	9	20	1	70
6.24	8	20	...	2	70
6.25	3	7
6.26	...	9	...	1	20	...	70
6.27	20	2	...	8	70
6.28	1	20	9	...	70
6.29	1	20	9	...	70
6.30	...	2	8

	3%	3½%	4%	4½%	5%	6%	7%
6.31	10	...	9	...	1	...	80
6.32	10	...	9	1	80
6.33	2	10	8	80
6.34	10	1	...	9	80
6.35	...	1	1	8
6.36	...	10	9	...	1	...	80
6.37	...	10	9	1	80
6¾	...	10	5	5	80
6.38	9	10	...	1	80
6.39	8	10	2	...	80
6.40	2	8
6.41	9	...	1	...	10	...	80
6.42	8	...	2	...	10	...	80
6.43	1	10	9	...	80
6.44	1	...	9	10	80
6.45	1	1	8
6.46	...	9	...	1	10	...	80
6.47	...	2	10	...	8	...	80
6.48	1	...	10	...	9	...	80
6.49	1	...	9	...	10	...	80
6.50	2	8



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